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THE APRIL MEETING AT STANFORD UNIVERSITY

The three hundred forty-first meeting of the American Mathematical Society was held at Stanford University on Saturday, April 3, 1937. The sessions were presided over at various times by Professors H. F. Blichfeldt, E. T. Bell, G. C. Evans, and W. A. Manning.

The attendance, about fifty in number, included the following thirty members of the Society:

H. M. Bacon, E. T. Bell, B. A. Bernstein, H. F. Blichfeldt, H. G. Brown, F. W. Dresch, John Dyer-Bennet, G. C. Evans, A. L. Foster, E. C. Goldsworthy, J. W. Green, O. G. Harrold, M. A. Heaslet, M. R. Hestenes, Glenn James, R. D. James, S. H. Levy, Hans Lewy, A. T. Lonseth, Dorothy Manning, W. A. Manning, Kenneth May, A. D. Michal, C. B. Morrey, T. M. Putnam, Pauline Sperry, Alvin Sugar, A. E. Taylor, J. M. Thompson, R. K. Wakerling.

Luncheon for members and their guests was served at the University Union.

Titles of papers read at the meeting follow. Those whose abstract numbers are followed by the letter *t* were read by title. Mr. Hall was introduced by Professor Bell; Messrs. Arnold, Elconin, Hyers, and Paxson by Professor Michal; Miss Manning by Professor Manning; Mr. Dresch and Mr. Shephard by Professor Evans; and Mr. Olds by Professor Uspensky.

1. *Concerning the convexification of continuous curves.* I, by Dr. O. G. Harrold, Jr. (Abstract 43-3-218.)

2. *Remarks on Nicod's reduction of Principia Mathematica*, by Professor B. A. Bernstein. (Abstract 43-3-219.)

3. *On binary quadratic forms with a single class in each genus*, by Mr. N. A. Hall. (Abstract 43-3-220.)

4. *The abstract theory of analytic functions and certain special cases*, by Dr. A. E. Taylor. (Abstract 43-3-221.)

5. *Reducible ternary arithmetical cubics*, by Professor E. T. Bell. (Abstract 43-3-222.)

6. *On simply transitive groups with transitive Abelian subgroups of the same degree.* II, by Miss Dorothy Manning. (Abstract 43-3-223.)

7. *A set of independent postulates for a group with equality undefined*, by Mr. H. A. Arnold. (Abstract 43-3-225.)

8. *Quotient differentials*, by Mr. Victor Elconin. (Abstract 43-3-226.)

9. *Arithmetic prime-decomposition and Boolean algebra*, by Dr. A. L. Foster. (Abstract 43-3-227.)

10. *A note on linear topological spaces*, by Mr. D. H. Hyers. (Abstract 43-3-228.)

11. *The general equilibrium and a simplified economic system*, by Mr. F. W. Dresch. (Abstract 43-3-229.)

12. *A problem in additive number theory*, by Professor R. D. James. (Abstract 43-3-230.)

13. *A further existence theorem for the ordinary differential equation in a linear topological space*, by Mr. E. W. Paxson. (Abstract 43-3-232.)

14. *On diophantine approximations*, by Professor H. F. Blichfeldt. (Abstract 43-3-233.)

15. *Quadratic functionals in the calculus of variations*, by Dr. M. R. Hestenes. (Abstract 43-3-234.)

16. *Problems of the incidence of taxation in a simplified economic system*, by Mr. R. W. Shephard. (Abstract 43-3-235.)

17. *The remainder in the approximate evaluation of the probability in the symmetrical case of James Bernoulli's theorem*, by Mr. C. D. Olds. (Abstract 43-3-236.)

18. *Properties of the quotients of the parameters in Fermat's equation*, by Professor Glenn James. (Abstract 43-3-237.)

19. *Waring's problem for a set of non-negative integers related to the polynomial* $m(x^3-x)/6+x$, by Dr. Alvin Sugar. (Abstract 43-3-238.)

20. *On the expansion of the remainder in the open-type Newton-Cotes quadrature formula*, by Dr. O. G. Harrold, Jr. (Abstract 43-3-239-*t*.)

21. *Polynomial and semi-polynomial forms of the confluent hypergeometric functions of two variables*, by Mr. N. A. Hall. (Abstract 43-3-240-*t*.)

22. *Commutative one-dimensional linear transformations*, by Professor Clifford Bell. (Abstract 43-3-241-*t*.)

23. *Representations in certain pure forms of degrees higher than the second*, by Professor E. T. Bell. (Abstract 43-3-242-*t*.)

24. *Characteristic differential properties of abstract differentiable groups*, by Mr. Victor Elconin. (Abstract 43-3-243-*t*.)

25. *The principal differential*, by Mr. Victor Elconin. (Abstract 43-3-244-*t*.)

26. *Additive number theory for the general cubic polynomial* (preliminary report), by Dr. Alvin Sugar. (Abstract 43-3-245-*t*.)

27. *Differential geometry of an abstract topological group with Banach coordinates.* II, by Professor A. D. Michal. (Abstract 43-3-231-*t*.)

28. *Arithmetical properties of sequences in rings*, by Professor Morgan Ward. (Abstract 43-3-224-*t*.)

T. M. PUTNAM,
Associate Secretary

THE APRIL MEETING IN CHICAGO

The three hundred forty-second meeting of the American Mathematical Society was held at the University of Chicago on Friday and Saturday, April 9–10, 1937. This meeting was the largest spring meeting in Chicago in the history of the Society. There were about two hundred persons in attendance, among whom were the following one hundred fifty-five members of the Society:

A. A. Albert, G. E. Albert, C. B. Allendoerfer, J. V. Atanasoff, W. L. Ayres, R. W. Babcock, R. H. Bardell, I. A. Barnett, Walter Bartky, S. F. Bibb, M. T. Bird, K. E. Bisshopp, G. A. Bliss, Henry Blumberg, L. M. Blumenthal, C. P. Brady, H. R. Brahana, J. L. Brenner, F. L. Brooks, R. S. Burington, L. E. Bush, F. A. Butter, W. D. Cairns, H. H. Campaigne, Harold Chatland, E. W. Chittenden, R. V. Churchill, A. B. Coble, L. W. Cohen, A. H. Copeland, Max Coral, J. J. Corliss, A. P. Cowgill, D. R. Curtiss, J. E. Davis, W. M. Davis, L. E. Dickson, J. L. Doob, P. L. Dressel, W. L. Duren, E. B. Escott, H. P. Evans, H. S. Everett, G. M. Ewing, Morris Friedman, A. S. Gale, H. L. Garabedian, H. H. Goldstine, G. D. Gore, Cornelius Gouwens, L. M. Graves, Louis Green, V. G. Grove, M. B. Haberzetle, D. W. Hall, W. L. Hart, G. A. Herr, A. D. Hestenes, T. H. Hildebrandt, Ralph Hull, C. C. Hurd, M. H. Ingraham, Dunham Jackson, E. D. Jenkins, Fritz John, C. D. Jones, V. R. Jones, A. J. Kempner, J. H. Kenna, W. S. Kimball, Fred Kiokemeister, S. C. Kleene, W. C. Krathwohl, A. C. Ladner, E. P. Lane, R. E. Langer, Lincoln LaPaz, C. G. Latimer, Solomon Lefschetz, R. A. Leibler, M. I. Logsdon, Louis De La Salle, H. A. Luther, E. J. McShane, C. C. MacDuffee, M. L. MacQueen, Morris Marden, Mary C. Garvin, R. G. Mason, A. E. May, J. R. Mayor, C. W. Mendel, Karl Menger, E. W. Miller, W. E. Milne, W. B. Morgan, Marston Morse, E. J. Moulton, S. B. Myers, A. L. Nelson, I. M. Niven, E. A. Nordhaus, Rufus Oldenburger, A. C. Olshen, E. J. Olson, A. P. O'Mara,

E. D. Pepper, J. E. Powell, Tibor Radó, G. Y. Rainich, W. C. Randels, W. T. Reid, H. L. Rietz, R. F. Rinehart, W. H. Roever, A. E. Ross, W. E. Roth, O. K. Sagen, R. G. Sanger, A. C. Schaeffer, Henry Scheffé, M. E. Shanks, H. A. Simmons, R. G. Simond, M. F. Smiley, Burke Smith, E. R. Smith, F. C. Smith, W. N. Smith, R. C. Stephens, E. C. Stopher, E. B. Stouffer, Otto Szász, Gabriel Szegö, J. D. Tamarkin, H. P. Thielman, W. J. Trjitzinsky, P. L. Trump, Henry Van Engen, V. J. Varino, J. I. Vass, H. E. Vaughan, R. W. Wagner, H. S. Wall, L. E. Ward, G. C. Webber, K. W. Wegner, J. V. Wehausen, E. T. Welmers, George Whaples, O. L. Wheeler, F. B. Wiley, L. A. Wolf, M. C. Wolf, F. E. Wood.

On Friday morning three sectional meetings were held. The first was devoted to Geometry, the second to Probability and Algebra, and the third to Analysis. On Friday afternoon Dr. W. T. Reid of the University of Chicago delivered a Symposium Lecture on *Boundary value problems in the calculus of variations*. A general session was held on Saturday morning.

The dinner, which was held at Judson Court on Friday, was attended by one hundred fifty-one persons. Professor E. P. Lane of the University of Chicago acted as toastmaster. Professor C. C. MacDuffee of the University of Wisconsin spoke as the new Managing Editor of the Transactions, Professor W. E. Milne of Oregon State Agricultural College spoke as a representative of the western group, and Professor J. D. Tamarkin of Brown University discussed the International Congress of Mathematicians which is to be held in 1940 at Harvard University.

On Friday morning Professors L. M. Graves and L. W. Cohen presided in the Section for Geometry, Professor A. A. Albert in the Section for Probability and Algebra, and Dean R. W. Babcock and Professor H. L. Rietz in the Section for Analysis. Professors Marston Morse and G. A. Bliss presided at the Symposium Lecture. The chairmen of the General Session on Saturday were Professors Dunham Jackson and T. H. Hildebrandt.

The titles of papers read at the meeting follow. Those whose abstract numbers are followed by *t* were read by title. Papers numbered 1 to 14 were read before the Section for Geometry, those numbered 15 to 28 before the Section for Probability and Algebra, those numbered 29 to 41 before the Section for Analysis, and those numbered 42 to 69 before the General Session. Dr. P. O. Bell was introduced by Dean E. B. Stouffer, Mr. R. M. Thrall by Professor H. R. Brahana, Dr. D. G. Fulton by Pro-

fessor G. Y. Rainich, Mr. Olaf Helmer by Professor L. M. Graves, Mr. W. A. Dwyer by Professor M. A. Basoco, and Reverend W. C. Doyle by Professor Francis Regan. Paper number 17 was read by Professor H. P. Evans, paper number 24 by Dr. M. C. Wolf, and paper number 45 by Professor M. L. MacQueen.

1. *Set functions and measurability conditions*, by Dr. E. T. Welmers. (Abstract 43-3-163.)

2. *Relations between certain continuous transformations of sets*, by Miss R. G. Simond. (Abstract 43-3-210.)

3. *Uniform continuity in non-metric spaces*, by Professor L. W. Cohen. (Abstract 43-3-217.)

4. *On the problem of imbedding two abstract spaces in a third*, by Dr. H. E. Vaughan. (Abstract 43-3-211.)

5. *Point set operators and their interrelations*, by Dr. E. C. Stopher, Jr. (Abstract 43-3-190.)

6. *Concerning biconnected sets*, by Professor E. W. Miller. (Abstract 43-3-203.)

7. *Space fillers in the plane*, by Professor R. P. Baker. (Abstract 43-3-175-*t*.)

8. *On a generalization of Bonnet's theorem*, by Mr. C. D. Jones. (Abstract 43-3-181.)

9. *Systems of quadrics associated with a point of a surface*, by Mr. Louis Green. (Abstract 43-3-186.)

10. *Geometric characterizations in projective differential geometry of curved surfaces*, by Dr. P. O. Bell. (Abstract 43-3-216.)

11. *Rectifiability of arcs in Finsler space*, by Dr. S. B. Myers. (Abstract 43-3-202-*t*.)

12. *The characteristic function in the history of optical problems*, by Dr. Max Herzberger. (Abstract 43-3-207-*t*.)

13. *Meaning and function of a picture*, by Professor W. H. Roever. (Abstract 43-5-253.)

14. *Note on the characterization of pseudo r-spheric* (S_n) *sets* (preliminary report), by Professor L. M. Blumenthal. (Abstract 43-5-254.)

15. *Transformations of the Pearson type* III *distribution*, by Mr. A. C. Olshen. (Abstract 43-3-192.)

16. *Postulates for the theory of probability*, by Professor A. H. Copeland. (Abstract 43-3-188.)

17. *On the postulational basis of probability* (preliminary re-

port), by Professor H. P. Evans and Dr. S. C. Kleene. (Abstract 43-3-174.)

18. *Note on the ideals of cyclic algebras*, by Dr. Ralph Hull. (Abstract 43-3-185.)

19. *On certain rational transformations*, by Dr. A. E. Ross. (Abstract 43-3-198.)

20. *On certain matrices and their determinants*, by Professor W. E. Roth. (Abstract 43-1-93.)

21. *Note on determinants*, by Professor M. H. Ingraham. (Abstract 43-3-173.)

22. *Linear matrix functions*, by Mr. R. W. Wagner. (Abstract 43-3-187-*t*.)

23. *Multiple-valued functions in matrix space*, by Mr. R. W. Wagner. (Abstract 43-5-257.)

24. *Convergence of a sequence of linear transformations*, by Professor M. H. Ingraham and Dr. M. C. Wolf. (Abstract 43-3-172.)

25. *Metabelian groups and trilinear forms*, by Mr. R. M. Thrall. (Abstract 43-3-171.)

26. *Forms in n variables whose ranks equal n*, by Professor Rufus Oldenburger. (Abstract 43-3-191.)

27. *Stochastic processes depending on a discrete-valued parameter*, by Dr. J. L. Doob. (Abstract 43-5-248.)

28. *The asymptotic Waring problem for homogeneous polynomial summands*, by Mr. Harold Chatland. (Abstract 43-5-247.)

29. *A generalization of trigonometry*, by Professor H. P. Thielman. (Abstract 43-3-197.)

30. *Generalizations of the Cauchy integral formula*, by Dr. D. G. Fulton. (Abstract 43-3-182.)

31. *The derivative of a rational function*, by Dr. J. L. Brenner. (Abstract 43-3-199.)

32. *On continued fractions representing constants*, by Professor H. S. Wall. (Abstract 43-3-170.)

33. *Discontinuous solutions for the problem of Bolza in parametric form*, by Mr. M. F. Smiley. (Abstract 43-3-176.)

34. *Sufficient conditions for a discontinuous solution in the calculus of variations*, by Dr. G. M. Ewing. (Abstract 43-3-201.)

35. *A third order irregular boundary value problem*, by Mr. H. A. Luther. (Abstract 43-3-195.)

36. *The solution of boundary value problems in physics by*

means of the Laplace transformation. Part I. *A theory for establishing a solution for problems with vanishing initial conditions*, by Professor R. V. Churchill. (Abstract 43-3-184.)

37. *Questions of continuity of functions in infinitely many variables*, by Mr. Olaf Helmer. (Abstract 43-3-179.)

38. *On conformal mapping*, by Dr. M. E. Shanks. (Abstract 43-3-212.)

39. *On extensions of linear and bilinear transformations*, by Dr. H. H. Goldstine. (Abstract 43-3-169.)

40. *Applications of derivatives of line integrals*, by Professor W. S. Kimball. (Abstract 43-3-193.)

41. *Representation of Stieltjes integrals by infinite series.* II, by Professor Fritz John. (Abstract 43-5-249.)

42. *Symmetric matrices in a modular field*, by Professor A. A. Albert. (Abstract 43-3-196.)

43. *Quaternion rings*, by Professor C. G. Latimer. (Abstract 43-3-183.)

44. *The algebraic determination of the class of a Riemann space*, by Dr. C. B. Allendoerfer. (Abstract 43-3-208.)

45. *Asymptotic curves on a surface*, by Professors E. P. Lane and M. L. MacQueen. (Abstract 42-11-391.)

46. *Geometry of a surface in the neighborhood of a spine*, by Professor V. G. Grove. (Abstract 43-3-206.)

47. *Note on properties of invariance of orthogonal polynomials in a complex variable*, by Professor Dunham Jackson. (Abstract 43-3-177.)

48. *Conditional invariants*, by Professor G. Y. Rainich. (Abstract 43-3-180.)

49. *On the connection formulas and the solutions of the wave equation*, by Professor R. E. Langer. (Abstract 43-3-189.)

50. *Asymptotic theory of linear differential equations containing two parameters*, by Dr. C. C. Hurd. (Abstract 43-3-178.)

51. *Note concerning Jordan metrics*, by Professor L. M. Blumenthal. (Abstract 43-3-194-*t*.)

52. *First theory of homogeneous diophantine equations of degree two*, by Professor L. E. Dickson. (Abstract 43-5-246.)

53. *On the partial sums of Fourier series*, by Professor Otto Szász. (Abstract 43-3-200.)

54. *On the order of the partial sums of Fourier series*, by Dr. W. C. Randels. (Abstract 43-3-164.)

55. *Existence theorems for single integral problems in the calculus of variations*, by Professor E. J. McShane. (Abstract 43-5-251.)

56. *A note on non-associative algebras*, by Dr. Nathan Jacobson (National Research Fellow). (Abstract 43-3-167-*t*.)

57. *p-algebras of exponent p*, by Dr. Nathan Jacobson (National Research Fellow). (Abstract 43-3-168-*t*.)

58. *Cyclic relations in point set theory*, by Dr. E. C. Stopher, Jr. (Abstract 43-3-205-*t*.)

59. *On a theorem due to A. Mullikin*, by Professor E. W. Miller. (Abstract 43-3-204-*t*.)

60. *On certain fundamental identities due to Uspensky*, by Mr. W. A. Dwyer. (Abstract 43-3-165-*t*.)

61. *On certain pseudo-periodic functions* (preliminary report), by Mr. W. A. Dwyer. (Abstract 43-3-166-*t*.)

62. *On Garvin's R(n) function*, by Reverend W. C. Doyle. (Abstract 43-3-214-*t*.)

63. *Note on harmonic functions represented by a Daniell integral*, by Dr. R. S. Martin. (Abstract 43-3-215-*t*.)

64. *A direct expansion proof of sufficient conditions for the non-parametric problem of Bolza*, by Dr. W. T. Reid. (Abstract 43-3-213-*t*.)

65. *Some self-adjoint boundary value problems* (preliminary report), by Mr. J. W. Calkin. (Abstract 43-3-209-*t*.)

66. *A significance test for component analysis*, by Professor P. G. Hoel. (Abstract 43-5-252-*t*.)

67. *Normal forms of convex regions under affine transformations*, by Professor Fritz John. (Abstract 43-5-250-*t*.)

68. *On Klein's oscillation theorem for boundary problems of the calculus of variations*, by Dr. W. T. Reid. (Abstract 43-5-256-*t*.)

69. *The theorem of Hildebrandt*, by Dr. H. H. Goldstine. (Abstract 43-5-255-*t*.)

M. H. INGRAHAM,
Associate Secretary

HADAMARD'S MEMORIAL VOLUME

Selecta. Jubilé Scientifique de M. Jacques Hadamard. Paris, Gauthier-Villars, 1935. 432 pp.

The task of selecting a group of representative papers from the writings of a mathematician as prolific and versatile as Hadamard is not an easy one. Obviously, papers containing fundamental results which have attracted almost universal attention should be included. On the other hand, a certain unity, showing a concept at different stages of development, is also desirable if such a volume is to prove useful as well as ornamental. It seems to the reviewer that the editors of this volume have succeeded admirably in their task, and have produced a book which not only gives an insight into the mathematical development of the author, but is also valuable as a source book and reference volume.

Since most of the papers have been in print for some time, only a brief survey of the volume is necessary. The papers have been roughly grouped under various headings, which give an indication of the variety of interests of the author, and his ability to make contributions in divergent fields. The first section is devoted to the Theory of Analytic Functions, and the first paper is, of course, his Paris doctoral thesis, entitled *Essai sur l'étude des fonctions données par leur développement de Taylor* (1892). It contains the introduction of the notion of greatest and least of the limits of a sequence utilized in the theorem now known as the Cauchy-Hadamard theorem for the radius of convergence of a power series, which is then applied to the study of conditions for the existence of a finite number of polar singularities on the circle of convergence. The thesis also includes sufficient conditions that the circle of convergence be a line of singularities. The third part of the thesis is not included. The second paper (1893) in this group, on entire functions, contains extensions of Poincaré's results on the relations between the growth of the coefficients, the distribution of zeros, and the order of entire functions. Of the three remaining papers in this set, mention might be made of the paper (1898) giving the relations of the singularities of the function $h(x)=\sum c_n x^n$ to those of $f(x)=\sum a_n x^n$ and $g(x)=\sum b_n x^n$ when $c_n=a_n b_n$. The last paper of this series, a meditation on the generalization of analytic functions (1912), anticipates the type of study of derivatives of functions of real variables fundamental to the theory of quasi-analytic functions.

The section devoted to the Theory of Numbers is number-theoretic mainly by implication. It centers in two studies of the Riemann zeta function, the first of which (1896) demonstrates that $\zeta(s)$ has no zeros on the line $\Re(s)=1$, and shows that the same methods are applicable to certain Dirichlet series. The second (1928) applies the formula generalizing Parseval's theorem relative to two developments in Dirichlet series to the function $\zeta'(s)/\zeta(s)$. The third paper in this set is the famous paper on the maximum value of a determinant (1893).

The next section headed Real Functions contains a single paper, *Sur les*

transformations ponctuelles (1906), concerned with the question of inversion of the system $X_i=f_i(x_1, \cdots, x_k)$, $(i=1, \cdots, k)$, x and X ranging over euclidean space of k dimensions, which calls attention to the need of conditions sufficient to insure not only the uniqueness, but the existence of an inverse.

In the division entitled *Differential Equations and Partial Differential Equations*, the first paper is really concerned with a study of what happens to a given curve near the origin under iteration of a system of functional equations $x_1=f(x, y)=ax+by+\cdots$, $y_1=g(x, y)=cx+dy+\cdots$, the roots of the characteristic equation $(a-s)(d-s)-bc=0$ being real and distinct. This paper is connected with differential equations only in that this problem arises in a study of Poincaré on the periodic solutions of differential equations. The next paper, *Sur les surfaces à courbures opposées et leurs lignes géodésiques* (1898), contains his classic results concerning geodesics on surfaces of this type without singularities in finite euclidean space and extending necessarily to infinity. He applies the Poincaré analysis situs in order to obtain a relation between the connectivity of the surface and the number of closed trajectories, probably the earliest suggestion of the effectiveness of analysis situs methods in the calculus of variations. The other two papers in this section devoted to partial differential equations center in the contrast between the Cauchy or initial condition problem, and the Dirichlet or boundary value problem, the first suggesting that a "sensible" question in mathematical physics leads to a problem with possibility of solution, the second paper pointing out that the statements of existence of solutions for the Cauchy type of problem must be examined carefully; for example, in the equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=\frac{\partial^2 u}{\partial t^2}$$

the Cauchy problem is possible of solution if t is the principal variable, but not always when x, y, or z is the principal variable.

In the part devoted to Calculus of Variations and the Calcul Fonctionnel, one finds, of course, the paper giving the first solution of the problem of determining the most general linear operation in the space of continuous functions on a finite range, superseded to be sure by Riesz's beautiful result, but still suggestive. The remaining papers include an outline of a direct method for proving the existence of a minimum for the simplest problem of the calculus of variations, a study of the stationary point of a function $f(x, y, z)$ on a curve $g_1(x, y, z)=g_2(x, y, z)=0$, and the determination of some special solutions of the functional equation

$$\frac{\partial}{\partial\alpha}\psi(A, B, \alpha)=-\int \nu(s, \alpha)\psi(A, M, \alpha)\psi(B, M, \alpha)ds$$

associated with Green's function.

The Geometry section includes two papers of an elementary character, one a note from his *Leçons de Géométrie* (1898), giving good sound advice on the analysis and solutions of geometric problems and demonstrations, and a paper showing how to determine directly the double points of a curve given in implicit parameter form, $f_1(x, y, t)=f_2(x, y, t)=0$. There is also the note from Tannery's book on function theory, in which the Kronecker index is used to

deduce from the Jordan curve theorem the Schoenflies theorem that if the continuous single-valued transformation $X=f(x, y)$, $Y=g(x, y)$, of the circle $x^2+y^2=1$ satisfies the condition that $f(x, y)=f(x', y')$, $g(x, y)=g(x', y')$ implies $(x, y)=(x', y')$ in and on this circle, then the transformation of the interior of the curve C corresponding to $x^2+y^2=1$ is one to one, and links it with the Brouwer fixed point theorem. The closing paper is devoted to a portion of géométrie anallagmatique, the geometry associated with inversions relative to a sphere.

The final section containing the most recent papers is devoted to the subject of Hydrodynamics, and most of these papers center in questions growing out of the principle of Huyghens stated in the form "from the state of the universe at the instant t_0 one can deduce the state at a later instant t', the deduction being possible by passing from t_0 to t_1 (t_1 being any instant such that $t_0<t_1<t'$) and from t_1 to t'." The papers are concerned largely with questions relating to the solutions of partial differential equations of the second order with boundary conditions of the mixed type, that is, conditions of initial or Cauchy type relative to one surface S, and of Dirichlet type along another surface Σ. The second paper is largely geometrical, centering in the contact between characteristics and the caustic curves derived from a system $z=f(x, y, \alpha, \beta)$ with two parameters. These results are applied in the third paper to study the solutions of partial differential equations.

A list of Hadamard's publications by years brings the volume to a close. The book as a whole leaves the impression that the editors have succeeded in producing a memorial volume worthy of the name, giving on the one hand an insight into the development of Hadamard's mathematical ideas, and, on the other, collecting in convenient form a group of papers which have played such an important role in the development of many phases of mathematics in the last forty years. In addition to their mathematical value, many of the papers of this group are models of clear exposition, worthy of emulation.

T. H. HILDEBRANDT

FORD ON ASYMPTOTIC DEVELOPMENTS

The Asymptotic Developments of Functions Defined by Maclaurin Series. By Walter B. Ford. University of Michigan Studies, Scientific Series, vol. XI. University of Michigan Press, Ann Arbor, 1936. viii+143 pp.

The subjects of mathematical monographs of the present day are all too often intelligible only to the initiates of their immediate fields. By and large the problems which are enunciable in few and simple terms, and which yet are neither classical nor beyond the reach of analysis, are rare. The problem to which the present book is devoted is such a one. It is, in brief, the following: A function of the complex variable being given through the medium of its Maclaurin series, say

$$f(z) = \sum_{n=0}^{\infty} c_n z^n,$$

to determine its nature and structure in the vicinity of the point $z = \infty$. This problem, though a relatively narrow one to be sure, is one of primary importance, not only in the general field of the theory of functions of a complex variable, but also, as the book at hand amply proves, in the theory of differential equations of the Fuchsian type.

The problem is one of very substantial and inherent difficulty, and much research must needs be done before it will have been solved with any show of completeness. The inroad upon it which is here presented has led to excellent results. In its main outlines the method of attack is the familiar one through the calculus of residues, and the analysis centers chiefly upon the evaluation or appraisal of contour integrals. It is basically assumed that the Maclaurin coefficients c_n depend upon n in the manner $c_n = g(n)$, where $g(w)$, regarded as a function of the complex variable w, fulfills certain conditions of analyticity and order of magnitude in appropriate regions of the w plane. As one would expect, the description of $f(z)$ in the region about $z = \infty$ in general calls for the use of asymptotic representations. Since the domains of validity of such representations are always restricted, the determination of both the domains and the representations valid within them is of necessity carried out. This is done in particular for the case in which c_n is any rational function of n, for the case of functions of so-called exponential type, those in which $c_n \sim h/\Gamma(n+p)$ with h and p any complex constants, and for the functions which are designated as of the Bessel type, those in which

$$c_n \sim \frac{h}{\Gamma(n + p_1)\Gamma(n + p_2)}.$$

As an illustration of the theory of the latter, the asymptotic forms of the Bessel functions themselves are obtained, incidentally without any use being made of their differential equation.

These general function theoretic deductions occupy the first seven chapters of the book. The material is excellently organized and is presented with all the clarity it permits. The earlier chapters are to be regarded as largely expository, though not entirely so, the principal source of material being the extensive re-

searches of E. W. Barnes. The later chapters present results of the author's own research which in the main have not been published heretofore.

Chapter 8, the final chapter of the book, will be found quite disjunct from the earlier work. The focus of the investigation is transferred here from the field of the theory of functions to the field of Fuchsian differential equations, specifically to the differential equation

$$z^2 p_0(z) \frac{d^2y}{dz^2} + z p_1(z) \frac{dy}{dz} + p_2(z) y = 0,$$

in which the functions $p_j(z)$, $j=0, 1, 2$, are quadratic polynomials, subject to the condition that the finite singular points of the differential equation be all regular. The classical general theory of this equation establishes the existence, in connection with each singular point, of a pair of solutions with specific indices. It does not, however, supply any method for relating the solutions associated with different singular points, and such relations have, in fact, been derived heretofore only for special differential equations, notably the hypergeometric equation. A theory of considerable generality bearing upon this important point is here developed. If the attention is fixed upon a solution relative to the point $z=0$, its series coefficients $c_n = g(n)$ are, of course, given by a certain recursion formula which is fixed by the differential equation. This formula is here regarded as a difference equation of which the function $g(w)$ is a solution. For the study of $g(w)$ the modern theory of difference equations is drawn upon to a considerable extent. Once its functional structure has been determined the results of the earlier chapters become available, and from them the representations of the solution in question in the vicinity of the point $z=\infty$ are obtained. A considerable differentiation into cases is necessary and is carried out, the form of the solutions depending, as is familiar, largely upon such items as the regularity or irregularity of the point $z=\infty$, whether or not the indices associated with this or that singular point do or do not differ by an integer, and so on. The material of this chapter is again practically all due to the author.

In general appearance and typographical style the book closely resembles the author's well known *Studies on Divergent Series and Summability* which appeared in 1916 in the same series. The book is thorough and serious, and the exposition is good. It is not to be inferred from this, however, that the text reads easily, for, in the author's own words, "the subject inherently calls for complicated forms of expression, and the proofs involve a regrettable amount of heavy analysis." At appropriate points, directions for further research are indicated. This reviewer feels that thanks are due Professor Ford for a significant work.

A list of errata, mainly furnished by the author, is appended.

Page 2, equation (7). Replace (z) by $f(z)$.

Page 12, equation (46). Replace $\sqrt{z}$ by $\sqrt{-z}$.

Page 14, 3d line beneath (51). Omit the words "except those."

Page 17, equation (4). Replace $i(\psi\}-\pi)$ by $i(\psi-\pi)\}$.

Page 22, line 5 in §6. Replace (3) by (5).

Line 6 from bottom. Replace (3) by (5).

Page 23, equation (4). Replace (-1) by $(-1)^n$.
Page 33, 1st line above (19). Replace (11) by (12).
Page 52, relation (61). Replace $\frac{d}{dw} P(w)$ by $w \frac{d}{dw} P(w)$.
Page 68, 7th line of (*b*). Replace $e^z z^{-1p}$ by $e^z z^{1-p}$.
Page 74, equation (3). Replace $\Gamma(z+a_2)/2)$ by $\Gamma(\ (z+a_2)/2)$.
Page 75. Put } at end of third line.
Page 78, 5th line. Omit = after $\frac{2^{2z-1}}{\sqrt{\pi}}$.
Page 78, 1st line above series expression for $h(t)$. Replace 22 by 25.
Page 80, 5th line below equation (35). Replace arg z by arg t.
Page 93, 4th line below Fig. 9. Replace $\geqq$ by $\leqq$.
Page 105, 14th line. Replace $f_0(x)$ by of $f_0(x)$.
Page 110, 1st line beneath (82). Replace V by γ.
Page 134, 4th line of 2. Replace $R(z)>0$ by $-\pi<\arg z<0$; also replace "section" by "sector."
In 5th line replace $R(z)<0$ by $0<\arg z<\pi$.

RUDOLPH E. LANGER

LEVY AND ROTH ON PROBABILITY

Elements of Probability. By H. Levy and L. Roth. Oxford, Clarendon Press, 1936. i+196 pp.

This book pretends to be "no more than an elementary treatment," and so makes no effort to cover all the many applications of probability, or even that part of statistics which is probability. It is not an elementary text in the sense in which American authors use the term, but it does begin at the beginning, and makes no mathematical demands on the reader beyond a knowledge of the ordinary calculus and some finite differences.

The first five chapters have to do with the usual matters, definitions of probability, arrangements, Bernoulli's theorem, and what the authors occasionally call the normal law, but usually refer to as the Gaussian law, although it is now known that the credit for it does not belong to Gauss. There is in Chapter VI an attempt at a rigorous presentation of an extension of the definition of probability for discrete numbers to continuous distributions; but surely it cannot be made wholly rigorous without more restrictions on the curve than that it be continuous. The net result is that the probability that a point lie on a certain arc is proportional to the length of that arc. It would seem therefore that this result might better be taken as a definition, for, as it is, one gets the impression that the length of an arc has something to do with the number of points it contains, which is not true, and of course is not an inference which the authors would like to have made. In Chapter VIII various forms of probability distributions are considered, including the Poisson distribution and those represented by the use of Hermite's polynomials.

The last chapter (IX) has to do with "scientific induction." Much of this chapter is too condensed for most readers unless they are familiar with the

substance already. It appears to be the culmination toward which the book is working, but on the whole it seemed to the reviewer not so carefully done as the rest. Probability is a tricky subject. An author must be careful in his use of language or else his own words may lead even him astray to say nothing of what they may do for his readers. Now the masterly introductory chapter and the carefully written second chapter on the meaning of probability are not followed by a similarly accurate use of words in this last chapter, perhaps because it seemed less necessary after the authors' ideas had been made clear initially; but there is no part of probability where precision of statement is more necessary than in the discussion of inference. In Chapter II, for example, the authors have stated that there are three kinds of probability which must be distinguished from each other: (1) mathematical probability (for example, the relative number of white balls in a bag); (2) statistical probability (the relative number of white balls that have been drawn from a bag); (3) a measure of psychological expectation (which is a subjective matter). These are discussed carefully and satisfactorily. On the other hand the last words of Chapter IX are not so satisfactory: "It is apparently three times more probable that the two populations are identical than that they are different." In which of these three senses does a hypothetical population have a probability, and if it does have a probability how can this afford a "scientific" basis for inference unless there is first established a relationship between the second kind of probability and the third? Moreover, the sentence just quoted concludes a discussion which apparently approves the use of that very disturbing assumption which has led most authors to discard Bayes' theorem, and which our authors also have condemned in a preceding discussion. There are other vague words and phrases in the chapter, such as (our italics) on page 184: "Suppose that a series of measurements is made of a quantity which in *normal* circumstances would have the value of m." · · · "There are n numbers · · · whose average is x; m is the mean *to be anticipated* if n were of infinite extent and if no factor had operated *to disturb the equilibrium* of the series."

Three tests of significance close the chapter: Fisher's t test, Irwin's test (1935), and Jeffrey's test (1935). The mathematical concepts behind these are given, and the reader is therefore at liberty to evaluate them as he likes, and by reading between the lines he can see, for example, that the tests of Irwin and Jeffrey, instead of being rival ways of looking at essentially the same thing, are somewhat similar ways of looking at different things, but it would be better if it were not necessary to read between the lines. There is room here for critical analysis. The early chapters had led us to expect that it would be keen and searching, but it is missing altogether. Instead, the remarks of the authors are quite casual and may give the impression that here are some tests that have some standing in the literature and that if an investigator does not fancy one of them he may try another. There are several instances throughout the book in which it should be, but is not, stated that the sampling is made one at a time with replacements (see example 1, page 160). This error seems to be due to the fact that in the statement of the Bernoulli theorem (page 58) the authors spoke of n objects as being "defined" as members of a population instead of being selected independently with replacements from it. Perhaps the idea of independence was implicit in the word "defined," but it was not expressed ex-

plicitly and so has not been copied in the applications to sampling. The word "exclusive" (page 52) is used in a peculiar sense in the statement of the fundamental product theorem, or else the language is otherwise faulty; for, if the classes were exclusive in the sense that an object could not belong to both, the probability of that object belonging to both would be zero. The table of normal areas is given in terms of multiples of $\sigma/\sqrt{2}$ as an argument instead of σ, as is now generally preferred.

A good deal of attention is paid (Chapter IX) to the problem of what happens to the distribution of observations when the experimenter "sweeps into his reading at say t_1 a number of readings at $t_1 \pm 1, t_1 \pm 2, \cdots$, and he "acts on the assumption that he is obtaining correct data in the given value of t" (page 147). It is further shown that a discussion of this problem is related to Bayes' theorem. The reviewer thinks that both these matters would be made much clearer if the same letter t were not used both for the true value being observed and for the reading which is being recorded. Using t for the first of these values and u for the second, the theories may be presented in the form of a two-way table, as follows:

u \ t	a			t			b	TOTALS
u				$f\phi$				$F(u)$
TOTALS				$f(t)$				1

Here $f\phi = f(t)\phi(t,u)$, and $F(u) = \int_a^b f(t)\phi(t,u)du$, which corresponds to the authors' expression (page 156)

$$U(t) = \int_a^b V(t+x)p(x)dx.$$

When the theory is thus presented, further applications are also immediately obvious.*

Besides its highly interesting and for the most part carefully written text, the volume contains in addition a large number of ingenious problems and illustrative examples.

BURTON H. CAMP

* The reviewer plans to discuss this in more detail in another paper.

SHORTER NOTICES

Über die Darstellung von Gruppen in Galoisschen Feldern. By R. Brauer. (Actualités Scientifiques et Industrielles, No. 195.) Paris, Hermann, 1935. 15 pp.

A given finite group can be represented in a number of ways as a linear group. The number of distinct representations of this sort when the coefficients of the linear substitutions are complex numbers has been shown by Frobenius to be the same as the number of conjugate sets of elements of the group. The author considers the same question, requiring the coefficients of the linear substitutions to lie in the minimal algebraically closed field determined by $GF(p)$, where p is a prime. When p does not divide the order of the group Dickson has shown that the number of representations is the same as for the field of complex numbers. When p divides the order of G the author shows that the number of distinct representations is the same as the numbers of conjugate sets of operators which have orders prime to p.

In the second part of the paper the author considers the number of indecomposable (unzerfällbar) components of the regular representation of the group and shows it to be the same as the number determined above, making use of the conjoint of the given group.

H. R. BRAHANA

Theoretical Astrophysics. By S. Rosseland. Oxford, Clarendon Press, 1936. 19+355 pp.

In the words of its author, the aim of this remarkable book is to formulate a program of theoretical astrophysics. It "contains what may be called a bird's eye view of our knowledge of stellar atmospheres and envelopes, projected on to atomic physics as a background." A second volume, dealing with the internal structure of stars, is promised by the author.

Nearly one-third of the book is devoted to developing those phases of quantum mechanics which are of interest in astrophysics. Particular emphasis is placed on spectroscopy. Even this portion of the book, which serves merely as an introduction to the central theme, is distinguished by clarity of presentation and elegance of mathematical argument, so much so, indeed, that the physicist not primarily interested in what follows will feel attracted by the manner in which the material is put.

The next chapter deals with the transfer of radiation through absorbing and emitting media, a problem which calls for an interesting application of hydrodynamic theory. After this the author turns to an investigation of empirical data on profiles and total intensities of stellar absorption lines. His survey is everywhere careful and critical, pointing out discrepancies as well as the numerous sources of inaccuracy of the theories involved. The opacity of stellar atmospheres is interestingly explained as chiefly due to absorption by excited electrons with accompanying photoelectric effect. Splitting and broadening of spectral lines due to Zeeman and Stark effects are treated rather

briefly; the various types of broadening and shift which are due to intermolecular forces and which have been of interest in the terrestrial laboratory are not discussed, presumably because of their absence in stellar spectra.

Several chapters are then devoted to the chemical constitution, to the dissociative equilibrium of stellar compounds, and to the temperatures of the stars, with special reference to the sun. After this the discourse again turns toward spectroscopy, the subject being that of forbidden transitions. Near the end of the book we find a discussion of evidence regarding the state of nebulae, their degree of ionization, their temperature, their chemical composition.

Every reader will close the book with an acute realization of the power of analysis which has shed considerable light on these most elusive problems, and of the vastness of the field which still waits for investigation. A book by Rosseland on astrophysics needs no recommendation. It is highly suitable that it should have appeared in the International Series of Monographs on Physics, thus aligning itself with some of the most outstanding treatises on modern physics.

HENRY MARGENAU

Éléments de géométrie infinitésimale. By G. Julia. Deuxième édition. Paris, Gauthier-Villars, 1936. 7+262 pp.

The second edition of Julia's differential geometry has entirely kept its original character. As such, it takes a very special position among the textbooks on the subject. In no other book are questions concerning contact and envelopes treated with so much elaboration and so much care. Not only does their discussion take more than a hundred pages of the first of the three chapters, but they also permeate many topics in later parts of the book. Great care is therefore taken in the application of the theory of implicit functions. In the new edition the author has here and there changed the order of the subjects, and added a few remarks, partly in footnotes, which have added to the clarity of an already very well written book.

D. J. STRUIK

Actualités Scientifiques et Industrielles. Exposés mathématiques publiés à la mémoire de Jacques Herbrand. Paris, Hermann.

1. *Sur quelques propriétés de polynomes.* By J. Dieudonné, 1934. 24 pp.
2. *Sur les suites stationnaires.* By N. Lusin, 1934. 19 pp.
3. *Étude des fonctions sousharmoniques au voisinage d'un point.* By Marcel Brelot, 1934. 55 pp.
4. *Charakterisierung des Spektrums eines Integral Operators.* By J. von Neumann, 1935. 20 pp.

1. The author develops an elementary method for determination of the radii of "starshapedness" and of "convexity" of some special classes of analytic functions and applies this theory to the case $z[P(z)]^{\alpha/n}$, where $P(z)$ is a polynomial of degree n and α is any real number.

2. Let (*) $E_0, E_1, \cdots, E_\alpha, \cdots, \alpha<\Omega$ be a transfinite sequence of Borel measurable sets, which is such that if α is a limit ordinal, then $E_\alpha=\lim_{\xi<\alpha} E_\xi$. The sequence (*) is said to be stationary if there exists a $\gamma<\Omega$ such that

$E_\gamma = E_{\gamma+1} = \cdots = E_\alpha = \cdots$, $\gamma \leqq \alpha < \Omega$. In generalizing some classical results of Baire the author proves that (*) is stationary if it is decreasing and if, in addition, each E_α is F_σ. The problem in the general case is unsolved.

3. This useful pamphlet consists of an Introduction followed by three chapters. In Chapter I the author gives a rapid but clear and rigorous survey of properties of convex and subharmonic functions, in some places giving new proofs or supplying missing details in older proofs existing in the literature. Chapter II contains the author's own investigation concerning the behavior of a subharmonic function in a neighborhood of an isolated singular point. The last Chapter, III, contains applications to the theory of harmonic functions and of solutions of the equation $\Delta u = \phi(P)$. The bibliographical references are numerous.

4. The operator $Af(x)$ defined over the space $L_2(-\infty, \infty)$ is said to be an integral operator if it can be represented in the form $\int_{-\infty}^{\infty} a(x, y)f(y)dy$ where $\int_{-\infty}^{\infty} |a(x,y)|^2 dy < \infty$ for almost all x. The main purpose of the author is to find necessary and sufficient conditions which have to be satisfied by a spectrum of a self-adjoint Hermitian operator in order that it be the spectrum of an integral operator. By a very elegant argument it is shown that necessary and sufficient conditions in question consist merely in the requirement that $\lambda = 0$ should be a limit point of the spectrum. An important place in the proof is occupied by the following theorem: *Being given any self-adjoint Hermitian operator H, it is always possible to find a completely continuous operator X of E. Schmidt's type of arbitrarily small norm, so that the spectrum of $H+X$ would reduce to a pure point-spectrum.* This theorem was proved by H. Weyl in the case of a limited operator H.

J. D. TAMARKIN

Introduction Mathématique aux Théories Quantiques. By Gaston Julia. Paris, Gauthier-Villars, 1936. vi+220 pp.

This book is the sixteenth of the well known series, *Cahiers Scientifiques*, and is the first of a series which proposes to give the mathematical foundation of quantum mechanics. In this first volume the essential difficulties of quantum mechanics (some of which concern the fact that Hilbert space is not finite dimensional) are merely foreshadowed, the attention being directed in the main to vector analysis in a space of finite dimensions. However, the treatment is sophisticated and designed, as far as possible, to carry over to the infinite dimensional case. Thus the treatment of linear equations is given in which the concept of rank is developed without the introduction of determinants. The derivation of the Jordan normal form for matrices is particularly clear, as is also the chapter on metric spaces. We can heartily recommend the book as an easy introduction to the well-known works of von Neumann (*Quantum Mechanics*), Stone (*Hilbert Space*), and Wintner (*Infinite Matrices*). There are only two parts of the book which seem to the reviewer other than highly commendable. The first of these contains the statement that unitary and Hermitian operators "se correspondent biunivoquement par les relations" (Cayley) $U = (H+i)/(H-i)$; $H = i(U+1)/(U-1)$. This is misleading since unitary matrices with a characteristic root unity are not cared for (except by a some-

what artificial limiting process). The correspondence $U=e^{iH}$ seems much more fundamental. The other point is in connection with the discussion of the values taken by the "quadratic" form associated with a matrix. The author seems unaware of the paper *On the Field of Values of a Square Matrix* by the present reviewer (Proceedings of the National Academy of Sciences, vol. 18 (1932)).

F. D. MURNAGHAN

The Rational Quartic Curve in Space of Three and Four Dimensions. By H. G. Telling. Cambridge, University Press; New York, The Macmillan Company, 1936. viii+78 pp.

This volume, Number 34 in the well known series of Cambridge Tracts in Mathematics and Mathematical Physics, is an introduction to the study of rational curves. The reviewer agrees that the best curve to select as representative of this type is the norm curve in four dimensions with its projections in ordinary space and in the plane. A detailed study of these rational quartics yields a wealth of geometric properties and of related configurations. On the other hand the analytic work involved is based on the algebraic theory of binary forms, and is not especially complicated for the quartic when expressed in the customary symbolic notation.

The present tract is condensed from a Fellowship Essay by the same author, and much of the material is here given as exercises. There are two chapters, the first of 40 pages on the norm quartic curve, and the second of 33 pages on the rational quartic in three dimensions. These are followed by several pages of notes on involutions on the curve. Only a few references are given in the text, but a selected bibliography is included which contains references to the extensive literature of the subject.

This little volume is well written, with excellent choice and arrangement of material. The author has produced a scholarly essay on a subject which richly deserves a place in this important series.

J. I. TRACEY

Idealtheorie. By W. Krull. (Ergebnisse der Mathematik, und ihrer Grenzgebiete, vol 4, No. 3.) Berlin, Julius Springer, 1935. vii+152 pp.

Krull's new book contains a very timely survey of the maze of material accumulated in recent years in the field of abstract ideal theory. Let it also be said to begin with that Krull's own fundamental contributions to the subject give him preeminent qualifications for the task.

The first concept of general ideal theory must be accredited to Dedekind. In his theory of the rings of all integers in fields of algebraic numbers one has the fundamental theorem that every ideal is a unique product of prime ideals. Dedekind also gives, however, consideration to rings in which this fundamental theorem is not true and where it has to be replaced by other decomposition theorems. Another introduction of general ideal theory came through Kronecker's theory of polynomial moduli. This theory was developed particularly by Lasker and Macauley, who showed its close relation with algebraic geometry. The impetus to the modern development came mainly through the work

of Emmy Noether who showed how the theory may be based upon simple axiomatic principles. Among the recent contributors one should mention, besides Krull, also van der Waerden for his extensive investigations on polynomial ideals and their geometrical applications.*

Krull limits his book to commutative ideal theory, and also omits essentially the papers connecting ideal theory with number theory. He begins with a short account of the properties of rings and moduli with operators. Then follows the general abstract ideal theory, the properties of prime and primary ideals, and the decomposition theorems. As a natural application follow the theory of polynomial ideals and the elimination theory for polynomials in several variables. The relation between ideal theory and algebraic geometry is considered extensively; let us mention only the theorems of Max Noether, Henzelt, Bezout, and the theory of multiplicities given by van der Waerden.

Among the further subjects covered in the book one may indicate the ideal theory of power series and of infinite algebraic fields. Another chapter deals with the ideal theory by finite algebraic extensions of the ring. This includes the theorems on norms and discriminants, giving generalizations of the results of the ordinary ideal theory of algebraic numbers. One of the most interesting sections gives Krull's own theory of absolute values, the relation of ideal theory to absolute values and metric properties of the ring and to the concept of integral elements.

Krull's book as a whole should not be classified as a textbook of ideal theory. To the beginner it will certainly present great difficulties. However, to the worker in the field it gives a general account which may be highly recommended. The complete bibliography further enhances its value.

OYSTEIN ORE

* Those interested in a further account of the foundation of ideal theory may be referred to my paper: O. Ore, *Abstract ideal theory*, this Bulletin, vol. 39 (1933), pp. 728–745.

NOTES

A new organization, known as the American Documentation Institute, has been incorporated on behalf of leading national scholarly, scientific, and informational societies to develop and operate facilities that are expected to promote research and knowledge in various intellectual fields. A first objective will be to develop and apply the new technique of microphotography to library, scholarly, scientific, and other material. The board of trustees elected consists of: Dr. R. C. Binkley, Western Reserve University; Dr. S. J. Buck, Director of Publications, National Archives; Watson Davis, Director, Science Service; Dr. J. T. Gerould, Librarian, Princeton University Library; Dr. Ludvig Hektoen, Chairman, National Research Council.

The Revista de Ciencias, official journal of the Faculty of Biological, Physical, and Mathematical Sciences of the University of San Marcos, Lima, invites American mathematicians to submit papers for publication.

The engineer N. Malaxa, of Bucharest, has presented a periodical award of 500 lei for the mathematical and physical sciences. The prize will be awarded by an international committee with headquarters at the N. Malaxa Institute in the science department of the University of Bucharest. The award will be made for the first time in June, 1938, and will be given to the young scientist who, in the opinion of the committee, has made the most important contribution to the problem of the resolution of equations in the period from January 1, 1934 to January 1, 1938.

The Royal Society of Edinburgh has awarded the Gunning Victoria Jubilee Prize for the period 1932–36 to Professor C. J. Darwin, master of Christ's College, Cambridge, formerly Tait professor of natural philosophy in the University of Edinburgh, for his "distinguished contributions in mathematical physics."

Professor John von Neumann, of the Institute for Advanced Study, has been elected to membership in the National Academy of Sciences, and Professor Oswald Veblen, also of the Institute for Advanced Study, has been elected a member of the council.

Dr. M. M. Slotnick, of the Humble Oil and Refining Company, Houston, Texas, has been elected Editor of Geophysics, the journal of the Society of Exploration Geophysicists.

Dr. Witwold Hurewicz, of Amsterdam, has been appointed professor of mathematics at the National University of Peking, Peiping, China.

Dr. Rudolph Peierls, at present assistant-in-research at the Mond Laboratory of the Royal Society at Cambridge, has been appointed to the newly established chair of applied mathematics at the University of Birmingham.

Assistant Professor A. A. Albert, of the University of Chicago, has been promoted to an associate professorship.

Dr. D. B. Ames, of the Rensselaer Polytechnic Institute, has been promoted to an assistant professorship.

Dr. Max Astrachan, of Antioch College, has been promoted to an assistant professorship.

Professor L. C. Bagby, of the Linsley Institute of Technology, Wheeling, W. Va., has been appointed Dean of the College of Engineering.

Dr. Samuel Borofsky, of Brooklyn College, has been promoted to an assistant professorship.

At the University of Cincinnati, the mathematics departments of the College of Liberal Arts and the College of Engineering have been combined. Professor Louis Brand, formerly head of the mathematics department in the College of Engineering, has been made head of the combined department.

Dr. R. H. Cameron, of the Massachusetts Institute of Technology, has been promoted to an assistant professorship.

Associate Professor L. P. Copeland, of Wellesley College, has been promoted to a professorship.

Professor H. T. Davis, of Indiana University, has been appointed to a professorship of mathematical statistics at Northwestern University.

Assistant Professor J. E. Davis, of Central Y. M. C. A. College, Chicago, has been promoted to an associate professorship.

Associate Professor J. H. Fithian, of Newark College of Engineering, has been promoted to a professorship.

Dr. B. C. Getchell, of Butler University, has been promoted to an assistant professorship.

Professor Harris Hancock, of the University of Cincinnati, has retired with the title professor emeritus.

Professor G. A. Harter, of the University of Delaware, has retired with the title professor emeritus.

Dr. M. A. Heaslet, of San Jose State College, San Jose, Calif., has been promoted to an assistant professorship.

Dr. M. R. Hestenes, of the University of California at Los Angeles, has been appointed to an assistant professorship at the University of Chicago.

Professor Alfred Hume, of the University of Mississippi, has retired with the title professor emeritus.

Dr. Ralph Huston, of Rensselaer Polytechnic Institute, has been promoted to an assistant professorship.

Dr. H. D. Larsen, of the University of New Mexico, has been promoted to an assistant professorship.

Dr. D. H. Lehmer, of Lehigh University, has been promoted to an assistant professorship.

Professor D. N. Lehmer, of the University of California, has retired.

Assistant Professor A. M. M. Lehr, of Bryn Mawr College, has been promoted to an associate professorship.

Associate Professor F. A. Lewis, of the University of Alabama, has been promoted to a professorship.

Professor W. R. Longley, of Yale University, has been appointed acting dean of freshmen.

Professor C. N. Moore, of the University of Cincinnati, has been made director of graduate studies in mathematics.

Professor C. A. Noble, of the University of California, has retired.

Assistant Professor C. O. Oakley, of Haverford College, has been promoted to an associate professorship.

Dr. W. T. Reid, of the University of Chicago, has been promoted to an assistant professorship.

Dr. Otto Szász has been appointed research lecturer in mathematics at the University of Cincinnati.

Assistant Professor H. S. Thurston, of the University of Alabama, has been promoted to an associate professorship.

Associate Professor H. S. Uhler, of Yale University, has been promoted to a professorship.

Assistant Professor Henry Van Engen, of Kansas State College, has been appointed head of the department of mathematics at Iowa State Teachers College, Cedar Falls.

Professor W. M. Whyburn, of the University of California at Los Angeles, has been appointed acting chairman of the department of mathematics to succeed Dr. E. R. Hedrick.

Assistant Professor H. N. Wright, of the College of the City of New York, has been promoted to an associate professorship.

The following appointments to instructorships are announced: University of Chicago: Dr. Saunders MacLane; Cornell University: Dr. J. W. Givens; New York University: Dr. J. C. C. McKinsey; Rutgers University: Dr. M. S. Robertson; University of Saskatchewan: Miss V. A. Ames; Spring Hill College, Spring Hill, Ala.: Rev. E. H. Larguier, S.J.; Yale University: Dr. Marshall Hall.

Mr. R. B. S. N. Ayyar, retired chief accountant of the Madras Port Trust, died January 17, 1937 at the age of sixty-one years. He was a foundation member of the Indian Mathematical Society.

Dr. S. F. Grace, senior lecturer in applied mathematics at the University of Liverpool and author of works on hydrodynamics and tides, died April 30, 1937 at the age of forty-one years.

Dr. Erich Trefftz, editor of the Zeitschrift für angewandte Mathematik und Mechanik, died on January 21, 1937.

President H. Y. Benedict, of the University of Texas, died suddenly on May 10, 1937 in his sixty-eighth year. He had been a member of the Society since 1901.

Professor W. I. Foster, of Northern Montana College, Havre, Mont., died on January 1, 1937.

Professor H. E. Slaught, since 1931 emeritus professor of mathematics at the University of Chicago, died on May 21, 1937 at the age of seventy-six years. He had been a member of the Society since 1899.

Dr. A. P. Wills, professor of mathematical physics at Columbia since 1909, died April 18, 1937 in St. Augustine, Fla. He was sixty-four years old.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

277. Mr. G. R. Kraus and Professor J. H. Neelley: *A classification of rational plane cubic curves.*

This paper gives methods of distinguishing the types of the cubic curve with special reference to the degenerate types. Using parametric notation the authors have shown how the different degenerate types may be detected and hence how each may be expressed. (Received May 25, 1937.)

278. Dr. J. H. Curtiss: *A note on the Cesàro method of summation.*

The following results are obtained: (1) If the sequences $\{S_{\lambda n+K}\}$ are respectively summable (C, α) to the values s_K, $K=0, 1, \cdots, \lambda-1$, for some $\alpha>0$, then the sequence $\{S_n\}$ is summable (C, α) to the value $(s_0+s_1+\cdots+s_{\lambda-1})/\lambda$. (2) If the series $\sum U_{\lambda n+K}$ are respectively summable (C, α) to the values u_K, $K=0, 1, \cdots, \lambda-1$, for some $\alpha>-1$, then the series $\sum U_n$ is summable to the value $u_0+u_1+\cdots+u_{\lambda-1}$. Examples are given to show that the theorems cannot be strengthened. (Received May 6, 1937.)

279. Dr. W. E. Sewell: *The derivative of a polynomial on various curves of the complex plane.*

Let $P_n(z)$ be a polynomial of degree n in $z=x+iy=re^{i\theta}$ and let $|P_n(z)| \leqq M$ on C. If C is an epicycloid or hypocycloid expressed in the usual parametric form with a divisible by b, then $|P_n'(z) \sin [a\phi/(2b)]| \leqq Mn/(2b)$, z on C. If C is a cardioid with polar equation $r=a(1-\cos\theta)$, then $|P_n'(z) \sin \theta/2| \leqq Mn/a$, z on C. If C is a rose curve with polar equation $r=a\cos k\theta$, then $|P_n'(z)| \leqq M(n+1)/a$, z on C. Of course these evaluations are independent of the position of the curves in the plane. (Received May 21, 1937.)

280. Professor D. V. Widder: *The successive iterates of the Stieltjes kernel expressed in terms of the elementary functions.*

The Stieltjes kernel is the function $H_1(x, y)=(x+y)^{-1}$. Its successive iterates are defined successively by $H_n(x, y)=\int_0^\infty H_1(x, t)H_{n-1}(t, y)dt$. It is proved in this note that $H_n(x, y)$ can be expressed in terms of the elementary functions. In fact it is a linear combination of a finite number of the functions

$[\log x-\log y]^{2k}/[(2k)!(x+y)]$ and $[\log x-\log y]^{2k+1}/[(2k+1)!(x-y)]$ with $k=0, 1, 2, \cdots$. The constants of combination are the coefficients in the Maclaurin expansion of $(\pi z/\sin \pi z)^n$. (Received May 7, 1937.)

281. Mr. O. H. Hamilton: *Fixed points under homeomorphisms of continua which are not connected im kleinen.*

It is shown that if M is a compact continuum in a metric space and if M contains no indecomposable continuum and contains no continuum which is the sum of two continua whose common part is disconnected, then every reversible continuous transformation of M into a subset of itself leaves some point of M fixed. It follows that if N is a compact continuum in the plane which contains no indecomposable continuum, does not separate the plane, and contains no domain, then every reversible continuous transformation of M into a subset of itself leaves some point of N fixed. Furthermore, if D is a compact simply connected domain in the plane whose outer boundary contains no indecomposable continuum, then every reversible continuous transformation of $\bar{D}$ into itself leaves some point of $\bar{D}$ fixed. An example is given of a compact acyclic continuous curve in the plane which cannot be carried into itself by any continuous transformation except the identical transformation. (Received May 15, 1937.)

282. Mr. V. G. Iyer: *On effective sets of points in relation to integral functions.*

Let $f(z)$ be an integral function of finite order ρ and type $\kappa(f)$. A sequence $[z_n]$ of distinct points having infinity as its sole limit point is said to be an effective set or E-set for $f(z)$ if, as n becomes infinite, $\limsup \log|f(z_n)|/|z_n|^\rho = \kappa(f)$. In this paper, the main problem is concerned with the determination of a sequence $[z_n]$ which forms an E-set for *each function* $f(z)$ of the class $C(\rho, d)$ of all functions of order ρ and type *less* than d, ρ and d being two positive numbers. It is shown that the zeros of the function $\sigma_\rho(z)=\prod_{n=1}^{\infty}(1-z^n/n^{\alpha n})$, where $\rho=2/\alpha$, form an E-set for the class $C(\rho, 1/\rho)$ from which an E-set can be derived for $C(\rho, d)$, d arbitrary, by means of an easy transformation. The idea involved in the paper was suggested by the fact that the lattice-points of the plane form an E-set for the class $C(2, \pi/2)$. (Received June 1, 1937.)

283. Professor N. H. McCoy: *Subrings of direct sums.*

In a joint paper with Deane Montgomery (abstract 43-3-135), it was recently shown that a commutative ring R, every element a of which satisfies the conditions $pa=0$, $a^p=a$, where p is a fixed prime, is isomorphic to a subring of a direct sum of $GF(p)$. The primary purpose of the present paper is to establish several theorems of this nature, at least one of them being a direct generalization of the theorem stated. (Received May 29, 1937.)

284. Miss Annita Tuller: *The measure of transitive geodesics on certain three-dimensional manifolds.*

It is the object of this paper to prove that almost all geodesics on certain three-dimensional manifolds are transitive. The metric $ds^2=(dx^2+dy^2+dz^2)/$

$(1-x^2-y^2-z^2)^2$ defines a hyperbolic geometry in the interior of the unit sphere S. The three-dimensional manifolds treated are those defined by considering as identical the points congruent to each other under certain groups of the rigid motions of this geometry, groups which are properly discontinuous in S but cease to be properly discontinuous on S. In particular, it is proved that almost all geodesics are transitive if the manifold is closed. This result is extended, however, to a more general case. Hopf (Mathematische Annalen, 1930, pp. 710–716) has shown that for certain dynamical systems almost all motions are either stable or unstable; stable in the sense that the motion returns arbitrarily close to any position formerly occupied, unstable in the sense that it stays outside any finite domain after a finite length of time. It is proved that if almost all geodesics are stable, then almost all of them are transitive. (Received June 2, 1937.)

285. Mr. W. H. Ingram: *On the numerical solution of integral equations.* Preliminary report.

Certain theorems on matrices due to Aitken, Duncan, and Collar are made the basis of a new method for the numerical solution of linear integral equations. Alternatively, the results can, in part, proceed from a result by Hilbert on the s-fold faltung of a bilinear form (Goettinger Nachrichten, 1906, p. 161). In the case of the Fredholm integral equation with symmetric kernel, the first characteristic function is given by the s-fold definite integral of $K(x, t_s)K(t_s, t_{s-1}) \cdots K(t_2, t_1)\xi(t_1)dt_1 \cdots dt_s$ where $\xi(t)$ is arbitrary and where s need not be greater than 4 for excellent accuracy in the case of the vibrating string, for instance. The corresponding characteristic number is given by the ratio of the s-fold to the $(s+1)$-fold such integral. Successive characteristic functions and numbers are obtained with kernels modified by the successive imposition of constraints eliminating the last mode. It would appear that the theory can be extended to non-symmetric kernels for which the characteristic numbers come in conjugate complex pairs. (Received June 5, 1937.)

286. Professor V. W. Adkisson: *Plane peanian continua with unique maps on the sphere and in the plane.*

The plane peanian continuum M is said to have a unique map on a spherical surface (or a plane) if and only if for any topological image M' of M on a sphere (or plane) S' and any topological image M'' of M on a sphere (or plane) S'' every homeomorphism of M' into M'' can be extended to a homeomorphism of S' into S''. The following theorems are proved. Theorem 1. The plane peanian continuum M has a unique map on a sphere if and only if one of the following conditions holds: (1) M consists of either a simple arc or triod, (2) M contains one cyclic element C which is a maximal triply connected cyclic curve of M, and $M-C$ consists of at most a countable number of arcs, $a_1, a_2, \cdots$, such that $\bar{a}_i \cdot \bar{a}_j = 0 (i \neq j)$ and each $\bar{a}_i \cdot C$ is a single point which lies on only one bounding circuit of C, provided that if C is a simple closed curve then $M-C$ is at most a simple arc. Theorem 2. The plane peanian continuum M has a unique map in the plane if and only if M is a simple arc, a triod, a simple closed curve, or if M contains a closed 2-cell C and $M-C$ consists of at most a countable number of

arcs $a_1, a_2, \cdots$, such that $\bar{a}_i \cdot \bar{a}_j = 0 (i \neq j)$ and each $\bar{a}_i \cdot C$ is a single point which lies on the only bounding circuit of C. (Received July 1, 1937.)

287. Professor A. A. Albert: *p-algebras over a field generated by one indeterminate.*

Let F be any perfect field of characteristic p, x be an indeterminate over F, K be algebraic over F. In particular the most interesting case is that where F is any finite field. It is shown that K is separable over $F(y)$ where y is the indeterminate whose p^eth power is x. The author considers division algebras of degree p^n over a centrum K as above and proves them all cyclic and with exponent equal to degree. Moreover all such algebras have a common splitting field $K(u)$, $u^{p^n} = y$. This paper will appear in full in the October number of the Bulletin. (Received July 2, 1937.)

288. Dr. Reinhold Baer: *Abelian fields and duality of abelian groups.*

E. Witt has recently characterized the finite abelian extensions of commutative fields by means of invariants. Witt's theory may be extended, almost without modification, to all those extensions whose group is abelian (finite or not). This may be done by the simple device of substituting Pontrjagin's duality theory for the classical theory of characters of finite abelian groups. (Received July 9, 1937.)

289. Dr. R. P. Boas: *Asymptotic relations for derivatives.*

One case of a well known theorem of Hardy and Littlewood states that if $f(x) \epsilon C^n(0, \infty)$, and if, as $x \to \infty$, $f(x) = o(x^r)$ and $x^n f^{(n)}(x) < O(x^r)$, $(r \geqq 0)$, then $x^k f^{(k)}(x) = o(x^r)$, $(1 \leqq k \leqq n-1)$. This is generalized, both by replacing $x^n f^{(n)}(x)$ by a more general linear differential form, and by replacing x^r by $\phi(x)$, a function of suitably regular behavior. (Results of the second kind overlap, but are not included in, more general results of Hardy and Littlewood.) In a later paper by D. V. Widder and the author, applications to the theory of the iterated Stieltjes transform will be given. (For this transform, see D. V. Widder, Proceedings of the National Academy of Sciences, vol. 23 (1937), pp. 242–244.) (Received July 7, 1937.)

290. Professor S. S. Cairns: *Normal coordinates for extremals transversal to a manifold.*

Consider a positive definite regular calculus of variations problem, $\int F(x, \dot{x}) dt$, where $F(x, r)$ is defined on an n-manifold R for $(x) = (x^1, \cdots, x^n)$ neighboring a point (x_0) and for $(r) = (r^1, \cdots, r^n) \neq 0$. In terms of *normal coordinates* with origin (x_0), the extremals having (x_0) for initial point are represented, in a certain neighborhood, by linear equations $y^i = c_i s$, $(0 \leqq s < s_0)$, where s is the arc length and the c_i are direction cosines. In this paper normal coordinates are obtained under weaker hypotheses than heretofore used. The author defines *normal coordinates* (z) *with respect to an* m*-manifold*, $(0 < m < n)$, M, on R. In terms of (z), M is defined, in a certain neighborhood, by $z^{m+1} = \cdots = z^n = 0$, and the general extremal cut transversally by M at its initial

point is defined by $x^i=a_i$ $(i=1, \cdots, m)$, $x^j=c_j s$, $(j=m+1, \cdots, n)$, $(0 \leqq s<s_0)$; wherein (a, c) are constants for each extremal, $c_j c_j=1$, and s is the arc length from M. As the c's vary over their $(n-m-1)$-sphere, the a's being fixed, one obtains the transversal extremals with a common initial point. They cover an $(n-m)$-manifold, differentiable save, in general, at the initial point. Necessary and sufficient conditions on $F(x, r)$ that this $(n-m)$-manifold be differentiable without exception for every m-manifold M are obtained. (Received June 1, 1937.)

291. Professor Leonard Carlitz: *An analog of the von Staudt-Clausen theorem.*

Put $t/\psi(t)=\sum B_m t^m/g(m)$, where $\psi(t)$ is the function defined in the Duke Mathematical Journal, vol. 1 (1935), p. 137, and $g(m)$ is a certain polynomial. The coefficients B_m have properties analogous to those of the ordinary Bernoulli numbers; some of these properties were discussed in the paper referred to. In the present paper some arithmetic properties of B_m are developed. The main result is that $B_m=G_m-e\sum 1/P$, where G_m is a polynomial, e an integer, and the summation is over a certain set of irreducible polynomials. The method of proof depends on certain ideas due to A. Hurwitz (Mathematische Annalen, vol. 51 (1899), pp. 196–226). (Received July 8, 1937.)

292. Professor R. V. Churchill: *The solution of linear boundary value problems in physics by means of the Laplace transformation.* II.

As an application of the theory in Part I (to be published in the Mathematische Annalen) the temperature distribution function is determined for a composite slab of finite thickness with zero initial temperature and variable surface temperature. The function is expressed in different real integral forms as well as in the form of a series, and conditions on the variable surface temperature are established under which these forms represent the solution of the boundary problem. The method of solution gives the Laplace transform of the temperature function, and by applying known Tauberian theorems to this transform some interesting properties of temperatures and heat transfer in composite slabs and bars are shown. (Received July 8, 1937.)

293. Professor H. S. M. Coxeter: *A graphical representation for the simple group of order* 504.

This paper concerns a group generated by three operators whose products in pairs are involutorial, as is also the product of all three (in a definite cyclic order). If m, n, p, are the periods of the three generators, the group is denoted by $G^{m,n,p}$. In particular, $G^{2,3,5}$, $G^{2,7,9}$, $G^{5,5,5}$, $G^{2,7,15}$ are the simple groups of orders 60, 504, 660, 12180, respectively. By regarding $G^{m,n,p}$ as a factor group of the group generated by inversions in three circles which form a curvilinear triangle of angles $\pi/2$, π/m, π/n, the author obtains a representation on a network of such triangles, in the manner that was first described by Dyck. By drawing new circles through the vertices where right angles occur, one derives a "semi-regular map" of m-gons and n-gons, in which the third generator of

$G^{m,n,p}$ appears as a "glide" along one of these circles. When the order of the group exceeds 500, the drawing is usually too extensive to be practicable; but the map for $G^{2,7,9}$ is particularly elegant. It consists of 84 triangles and 36 heptagons, and forms an unorientable surface of connectivity 8. (Received June 29, 1937.)

294. Dr. Aaron Fialkow (National Research Fellow): *The Riemannian curvature of a hypersurface.*

The author proves the following generalization of a well known theorem of Riemannian geometry. When none of the lines of curvature at a point of a V_n in a V_{n+1} are tangent to a null vector, the difference of the Riemannian curvatures of the V_n and V_{n+1} for the orientation determined by the directions of two lines of curvature at the point is numerically equal to the product of the corresponding normal curvatures; the sign is determined by the character of the normal to V_n in V_{n+1}. By means of this result, a characteristic property of flat spaces is derived. (Received July 8, 1937.)

295. Professor Tomlinson Fort: *The calculus of variations applied to Nörlund's sum.*

The principal solution of the linear difference equation $\Delta y(x)=f(x)$ which Nörlund calls a "sum" has many resemblances to an integral. The purpose of the present paper is to apply methods of the calculus of variations to the problem of minimizing such "sums" taken between constant limits. An analog to Euler's equation is obtained. There is also a treatment of the second variation with the development of further necessary conditions. (Received June 17, 1937.)

296. Dr. Bernard Friedman: *Analyticity of equilibrium figures of rotation.*

The possible forms of relative equilibrium of a homogeneous gravitating mass of liquid, when rotating about a fixed axis with constant angular velocity, are given by an integral equation (to be specified later) in which the domain of integration depends upon the equilibrium figure. Let the z-axis be the axis of rotation, ω the angular velocity, and R the region containing the rotating mass. Then at equilibrium the following equation holds: $(\omega^2/2)(x^2+y^2)-\int_R dV/\overline{PQ}$ = constant, where $P(x, y, z)$ is any point on the surface of R, and Q, the integration point, runs through R. Lichtenstein proved that, at all points where the apparent gravity is not zero, the surface possesses continuous derivatives of all orders. In this paper, it is proved that the surface is also analytic. The method is essentially that used by E. Hopf in proving the analyticity of solutions of elliptic differential equations of the second order. (Received June 22, 1937.)

297. Professor Edward Kasner: *The geometry of conformal symmetry (Schwarzian reflection).*

A general analytic curve C defines a symmetry transformation S which is a conformal covariant. Let C_1 be any curve going through a point P of C and let C_2 be the image of C_1 in C. In this paper fundamental theorems are obtained

stating relations between the curvatures γ, γ_1, γ_2 and their successive derivatives with respect to arc. By definition, C is the conformal bisector of the curvilinear angle C_1C_2. In particular if C_1C_2 is a horn angle, the quantities γ, $d\gamma/ds$, $d^2\gamma/ds^2$ assume mean values, but not the higher derivatives. (In the equilong theory *all* derivatives take mean values.) To be published in Annals of Mathematics, October, 1937. (Received July 3, 1937.)

298. Professor Edward Kasner: *The two conformal invariants of fifth order.*

Associated with a first order contact of two curves (horn angle H_2), there is a unique absolute conformal invariant of third order, involving curvatures and their first derivatives. (See Proceedings of the National Academy of Sciences, April, 1936.) If the contact is of second order (horn angle H_3), the unique conformal invariant is of fifth order. If two curves are orthogonal (general curvilinear right angle) an invariant, also of fifth order, results. The two invariants may be interrelated by the theory of symmetry (Schwarzian reflection). When one of the curves in H_3 is assumed to be a circle, one obtains Mullins' inversive invariant of a general curve. Relative invariants are also studied. (Received July 3, 1937.)

299. Professor D. H. Lehmer: *An application of Schläfli's modular equation to a conjecture of Ramanujan.*

The conjecture in question may be quoted as follows. Let $q=5$, 7, or 11. If n is chosen so that $24n-1$ is divisible by q^α, then the number $p(n)$ of unrestricted partitions of n is divisible by q^α. In the present paper the author shows how to use to advantage the so-called Schläfli's elliptic modular equation connecting $f(mz)$ with $f(z)$ (where $f((-D)^{1/2})$ is either of the two fundamental class-invariants associated with D), to the problem of verifying the above conjecture by means of the Hardy-Ramanujan series for $p(n)$ whose application for every n has been justified recently by Rademacher. The method is applied to four new cases in which it is proved that the numbers $p(n)$ for $n=1224$, 2052, 2474, and 14031 are respectively divisible by 5^4, 11^3, 5^5, and 11^4. All four results are in accord with the conjecture of Ramanujan. (Received June 28, 1937.)

300. Mr. Ingo Maddaus: *On completely continuous linear transformations.* Preliminary report.

Let E be a Banach space with the additional property that every element g is of the form $\int_0^1 f(t)d_t\lambda(g;t)$, where $f(t)$ is a continuum of elements of the space, each of unit norm, and $\lambda(g;t)$ is a bounded linear operation in g for each t. Then any completely continuous linear transformation on a Banach space to a space of type E is the strong limit of a sequence of linear transformations each of which transforms its domain into a space of finite dimension. (Received July 9, 1937.)

301. Mr. Philip Newman: *The maximum of a superharmonic function.*

A superharmonic function is known to take its minimum on the boundary.

By means of "harmonic improvement" and by forming the minimum function of a finite set of superharmonic functions, a sequence of functions can be constructed converging uniformly to a superharmonic function which takes its maximum on the boundary and has the same boundary values as the given superharmonic function. Employing Perron's boundary functions (Mathematische Zeitschrift, vol. 18 (1923), pp. 42–54) the foregoing theorem can be used to demonstrate that the upper and lower functions for a boundary value problem have a common limit function which is at once seen to be continuous and harmonic. (Received July 8, 1937.)

302. Professor Gordon Pall: *Applications of generalized quaternions to regularity of ternary quadratic forms.* Preliminary report.

Let $\lambda=1$, 2, or 3. Let n be a positive integer, $n\equiv 1 \pmod 8$ if $\lambda=1$ or 2, $n\equiv 1 \pmod{24}$ if $\lambda=3$. In the equation $n=\lambda x_1^2+x_2^2+x_3^2$ all the solutions in integers x_h satisfy either A or B, where A implies x_2 or $x_3\equiv 0 \pmod{4\lambda}$ for $\lambda=1$, 2 and 0 $\pmod{2\lambda}$ for $\lambda=3$; B implies x_2 or $x_3\equiv 2\lambda \pmod{4\lambda}$ for $\lambda=1$, 2, and $\equiv\lambda \pmod{2\lambda}$ for $\lambda=3$. If n is a square m^2, $m>0$, such that $(-\lambda\,|\,m)=1$, then all proper solutions (at least) are of type A; while if $(-\lambda\,|\,m)=-1$, all proper solutions are of type B. But if n is not a square, there are equally many solutions of each type A and B. The proof is based on properties of $\bar{\imath}at$, where a and t are quaternions in which $i^2=-1$, $j^2=-\lambda$, $\cdots$; and there are immediate applications to the proof that certain forms (for example, $x^2+3y^2+36z^2$) are regular though they belong to genera of more than one class. (Received June 15, 1937.)

303. Professor Gordon Pall: *The quaternion congruence* $\bar{\imath}at\equiv b \pmod g$ *and the equation* $h(8n+1)=x^2+y^2+z^2$.

If h contains no factor >1 of the form $8f+1$, then every solution of $h(8n+1)=x_1^2+x_2^2+x_3^2$ in coprime integers x_m belongs to one of two classes of residues $(x_1, x_2, x_3) \pmod{4h}$. If $8n+1$ is a square s^2, $s>0$, then all the solutions are in one class or the other according as $s\equiv 1$ or 3 (mod 4). But if $8n+1$ is not square, there are equally many solutions in each class. The proof depends on a study of the form $\bar{\imath}at \pmod g$ where a is a pure integral quaternion, t is an arbitrary integral quaternion, and g is a rational integer. If g is an odd prime dividing Na but not a, $\bar{\imath}at$ represents by choice of t exactly half the residues, besides (0, 0, 0), of norm $\equiv 0 \pmod g$; unless $g\equiv 1 \pmod 8$ the other half are represented by $\bar{\imath}a't$, where a' is obtained from a by permuting or changing signs of i, j, k. Similar results hold if g is composite. If b is a pure quaternion, and $Nb\not\equiv 0 \pmod 4$, all solutions t of $\bar{\imath}at\equiv b \pmod 8$ satisfy one and only one of $\bar{\imath}t\equiv 1$ or 3 (mod 4). Conditions for solvability with $g=2^n$ or p^n are obtained. (Received June 15, 1937.)

304. Professor W. V. Parker: *On symmetric determinants.*

In this paper the following results are obtained. If D is a symmetric determinant of order $n>4$ with real elements, in which all principal minors of some order $k>3$ and also all principal minors of order $k-3$ are zero, then D is

of rank $k-1$ or less. If $n>5$ and M is any principal minor of D of order $n-1$ and if the principal minors of D of orders k and $k-3$ are divided into two sets, first those which are minors of M and second those which are not minors of M; then if all minors in either set are zero, D is zero. (Received July 6, 1937.)

305. Professor W. V. Parker: *The characteristic roots of a matrix.*

If λ is a characteristic root of the matrix $A=(a_{ij})$ and if $R_i=\sum_{j=1}^{n}|a_{ij}|$, $T_j=\sum_{i=1}^{n}|a_{ij}|$, and $2S_i=R_i+T_i$, it is shown that the absolute value of λ does not exceed the greatest of the S_i. In case the matrix A is Hermitian, an upper limit for the characteristic root of least absolute value and a lower limit for the characteristic root of greatest absolute value are given also. (Received June 25, 1937.)

306. Dr. Moses Richardson: *On periodic transformations of complexes.*

Let K be an abstract simplicial complex and T a single-valued simplicial mapping of K on itself of period m which preserves incidence and dimension, where m is arbitrary. Let k be the complex obtained by identifying all elements of K which are congruent under the powers of T; let K^* be the subcomplex of K consisting of all invariant simplexes; and let k^* be the counterpart of K^* on k. Under certain combinatorial restrictions, the author determines the mod m Betti numbers of (a) k; (b) k mod k^*; and (c) K^*. For the case $m=2$, (a) and (b) have been discussed previously by P. A. Smith and the author. The method used is an extension of that of Smith. (Received July 3, 1937.)

307. Dr. J. B. Rosser: *The nth prime is greater than n* log *n.*

Counting 2 as the first prime, denote the nth prime by $p(n)$. For large n, $p(n)=n\log n+n\log\log n-n+O[(n\log\log n)/\log n]$, so that for large n, $p(n)>n\log n$. In this paper it is shown that for all positive n, $p(n)>n\log n$. As a consequence of this, one can prove that if B_n, C_n, and D_n are defined respectively for $n\geqq 2$ by $\sum_{m=1}^{n}1/p(m)=\log\log n+B_n$, $\prod_{m=1}^{n}(1-1/p(m))=C_n/\log n$, and $\prod_{m=2}^{n}(1-2/p(m))=D_n/\log^2 n$, then $B_n>B_{n+1}$, $C_{n+1}>C_n$, and $D_{n+1}>D_n$. It is readily shown that all three sequences approach limits, and the numerical values of these limits are given to ten decimal places. These results will be used in the author's paper, *An improvement of Brun's method in number theory*. For completeness, the following theorems are also proved. If $n>3$, $p(n)<n\log n+2n\log\log n$. If $n>1$, $n\log n+n\log\log n-n-9n<p(n)<n\log n+n\log\log n-n+9n$. (Received June 26, 1937.)

308. Dr. A. C. Schaeffer: *Existence theorem for the flow of an ideal incompressible fluid in two dimensions.*

Let initial velocity components be defined throughout a plane region at a given initial time such that the divergence is zero, the normal component of the velocity at the boundary is zero, and certain smoothness conditions are satisfied. It is shown that a flow exists for all time which satisfies at the initial time the given initial conditions. The method of successive approximations is used, which allows one to construct the flow for all time. (Received June 21, 1937.)

309. Dr. A. R. Schweitzer: *On a classification of metrical relations in the foundations of geometry.*

This paper is in continuation of two preceding papers reported in this Bulletin, abstracts 43-1-53 and 43-1-92. The basis of the author's classification of metrical relations is the analogy between (1) order and metrical relations, (2) metrical relations on different levels of measurement. In the first category the author places the metrical analogs $E^{(S)}$, $E^{(B)}$, $E^{(I)}$, $E^{(K)}$ of his relations S, B, I, K. The relation E expresses types of equidistance of points from other points; in particular, the relation $E^{(K)}$ expresses congruence between two n-dimensional $(n+1)$-ads of points $(n=1, 2, 3, \cdots)$. This analogy leads to an abstract empty bracket notation in which the relations S, B, I, K are replaced by empty brackets of the type "( )" which may be filled by $E^{(S)}$ or S, $E^{(B)}$ or B, $\cdots$, respectively. In the second category the author places as analogs of his relations, $\alpha_1\alpha_2\cdots\alpha_{n+1}=k\cdot\beta_1\beta_2\cdots\beta_{n+1}$ and $\alpha_{jn}=k\cdot\beta_{jn}$, the relations, $SP(\alpha_1\alpha_2\cdots\alpha_{n+1})=k\cdot SP(\beta_1\beta_2\cdots\beta_{n+1})$ and $SP(\alpha_{jn})=k\cdot SP(\beta_{jn})$ where SP denotes the oriented metrical linear, areal, $\cdots$ "span," and the n-dimensional angular "span," respectively. The coefficient k denotes a real number, and in the latter type of relations the $(n+1)$-adic terms need not be cospatial. For $n=1$ reference is made to Pieri and Minkowski. (Received June 30, 1937.)

310. Dr. A. R. Schweitzer: *A theory of congruence in the foundations of geometry.*

Supplementing the author's n-dimensional $(n=1, 2, 3, \cdots)$ oriented descriptive "K" systems outlined in previous reports in this Bulletin, sets of congruence axioms are constructed (1) primarily without, (2) with, reference to orientation. These systems are phrased in terms of undefined $(n+1)$-adic relations $E^{(K)}, K$ and $E_K^{(K)}, K$ respectively. This theory is shown to be equivalent to the well known congruence theory of Hilbert. In a second part of the paper a study is made of the analogy between the relations K and E based on the fact that both relations are effective between $(n+1)$-ads of points and both are formally of the nature of equality. This analogy leads to the construction of a general abstract theory $\mathcal{G}[K, (\ )]$, in terms of an empty bracket relation, which remains valid if the bracket is filled by K throughout or by E throughout. A congruence theory equivalent to the preceding theory is then obtained from $\mathcal{G}[K, (E)]$ by adding certain axioms in terms of K and E. In the dyadic case, only one (existential) axiom is required. This development represents on an absolute, axiomatic basis, the gradual transition from order to congruence in the foundations of geometry. (Received June 30, 1937.)

311. Mr. C. E. Springer: *On the metric differential geometry of a general surface in a linear space of four dimensions.*

This paper employs the tensor notation to extend the results of V. G. Grove (Transactions of this Society, vol. 39 (1936), pp. 60–70) to a general surface in S_4, using the differential equations of the surface in the form developed by Eisenhart (*Riemannian Geometry*, 1926, p. 189). Further, a study is made of the congruence which is determined by the line of intersection of the normal plane to the surface at a point and the plane determined by the two

lines of intersection of planes of certain degenerate quadrics. The condition that this congruence be conjugate to the parametric net on the surface is obtained. Two geometrically defined reciprocal cubic cones are studied at a point on the surface. The surfaces for which these cubic cones coincide at every point have the property that the congruence is conjugate to them. (Received July 9, 1937.)

312. Mr. J. V. Wehausen: *Bounded sets in topological linear spaces.*

This paper is concerned with bounded sets in a topological linear space satisfying the Axioms 1, 3, 4, 5, 6 of John von Neumann (Transactions of this Society, vol. 37 (1935), p. 4). In such a space a linear operation which is continuous takes bounded sets into bounded sets; the converse is proved provided there exists a bounded open set in the domain, this latter condition, however, being sufficient for the existence of an equivalent (F)-metric. If the space is also convex (von Neumann, loc. cit., Axiom 7), this condition is both necessary and sufficient for the existence of an equivalent norm metric. By defining a weak topology in a norm vector space by the neighborhoods $U(F_1, \cdots, F_n; \epsilon) = E_x[|F_i(x)| < \epsilon]$, F_i linear functionals on the space, it is proved that no neighborhood is bounded, the topology is not metrizable, and the space is of the first category unless there exists a finite total set of functionals (Banach, *Opérations Linéaires*, p. 42) for the space, this being equivalent to finite dimensionality. (Received July 8, 1937.)

313. Professor F. H. Safford: *Magic squares of the fifth order.*

This paper presents the computation of all squares which can be formed from the first twenty-five natural numbers where all rows and columns have the same sum and all pairs of numbers symmetrically located as to the center have another constant sum, thus superseding the usual diagonal condition. The number of these squares is 3004, all independent and transformable in 128 ways each. The squares are listed with respect to any two independent and non-central rows or columns, and the compilation has been deposited with the Department of Mathematics of the University of Pennsylvania. (Received July 8, 1937.)

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VOGEL (T.). See HERMANN (G.).

WOLTERECK (H.). See DEBYE (P.).

OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

STATE COLLEGE, PENNSYLVANIA, SUMMER MEETING AND COLLOQUIUM, September 7–10, 1937.

Abstracts must be in the hands of Associate Secretary T. R. Hollcroft, 531 West 116th St., New York, N. Y., not later than August 3. Professor John von Neumann will deliver a series of colloquium lectures on *Continuous geometry*. By invitation of the Program Committee, Professor Hassler Whitney will give an address entitled *Topological properties of differentiable manifolds*.

NEW YORK CITY, October 30, 1937.

Abstracts must be in the hands of Associate Secretary T. R. Hollcroft, 531 West 116th St., New York, N. Y., not later than October 2. Abstracts received by the Secretaries not later than September 3 will appear in the September issue of the Bulletin. By invitation of the Program Committee, Professor D. J. Struik will deliver an address entitled *The application of tensor analysis to problems of electrical engineering*.

IOWA CITY, IOWA, November 26–27, 1937.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wisconsin, not later than October 30; they will appear in the November issue of the Bulletin. By invitation of the program committee, addresses will be delivered by Professor W. J. Trjitzinsky on *Singular point problems in the theory of linear differential equations*, and by Professor A. H. Copeland on *Fundamental concepts of the theory of probability*.

PASADENA, CALIFORNIA, November 27, 1937.

Abstracts must be in the hands of Associate Secretary T. M. Putnam, University of California, Berkeley, California, not later than October 30; they will appear in the November issue of the Bulletin.

INDIANAPOLIS, INDIANA, ANNUAL MEETING, December 28–30, 1937.

Abstracts must be in the hands of Associate Secretary T. R. Hollcroft, 531 West 116th St., New York, N. Y., not later than November 23, 1937. Abstracts received by the Secretaries not later than November 2 will appear in the November issue of the Bulletin.

R. G. D. RICHARDSON, *Secretary of the Society*.

Articles for insertion in the BULLETIN should be addressed to the Managing Editor, D. R. CURTISS, Northwestern University, Evanston, Ill. Reviews should be sent to W. R. LONGLEY, Yale University, New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

Subscriptions to the BULLETIN, orders for back numbers, and inquiries in regard to non-delivery of current numbers should be addressed to the American Mathematical Society, 450–459 Ahnaip St., Menasha, Wis., or Low Memorial Library, 531 West 116th St., New York, N. Y.

The initial fees and the annual dues of members of the Society (see this BULLETIN, p. 322, May, 1930; and the List of Officers and Members, September, 1936, p. 67) are payable to the Treasurer of the Society, Professor P. A. Smith, Low Memorial Library, 531 West 116th St., New York, N. Y.

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